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Unbiased Tracking with Converted Measurements

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In many target tracking problems it is advantageous to perform tracking in a different coordinate system than the measurements. In these cases, the measurements require some form of conversion prior to use in tracking. There are two potential issues that arise when performing converted measurement tracking. The first occurs when the measurement conversion results in a biased (converted) measurement. The second is estimation bias that occurs when the estimate of the converted measurement error covariance is correlated with the measurement noise. First, previously proposed unbiased conversions are examined. Following this, the “decorrelated unbiased converted measurement” approach is examined and shown to overcome the issues of conversion bias and estimation bias. Finally this approach is evaluated in a Converted Measurement Kalman Filter (CMKF).

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Unbiased Tracking with Converted Measurements

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***Abstract*— In many target tracking problems it is advantageous to perform tracking in a different coordinate system than the measurements. In these cases, the measurements require some form of conversion prior to use in tracking. There are two potential issues that arise when performing converted measurement tracking. The first occurs when the measurement conversion results in a biased (converted) measurement. The second is estimation bias that occurs when the estimate of the converted measurement error covariance is correlated with the measurement noise. First, previously proposed unbiased conversions are examined. Following this, the "decorrelated unbiased converted measurement" approach is examined and shown to overcome the issues of conversion bias and estimation bias. Finally this approach is evaluated in a Converted Measurement Kalman Filter (CMKF).**

I. Introduction

In some estimation problems measurements are converted to a different coordinate system prior to tracking. If this conversion process results in a biased converted measurement, degradation in tracking performance occurs. Use of unbiased measurement conversion is preferred for these cases. In this paper previous work on unbiased converted measurements, including the recently proposed Decorrelated Unbiased Converted Measurement (DUCM) [2], are analyzed for radar tracking scenarios. The conversion techniques are then evaluated in a Converted Measurement Kalman Filter (CMKF). In this filter the polar measurements of range, r, and bearing, α, are converted to Cartesian coordinates to allow for the use of a linear Kalman filter. The conventional conversion is shown below:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r\cos\alpha \\ r\sin\alpha \end{bmatrix} \tag{1}$$

The polar to Cartesian conversion equations include a trigonometric function of a random variable. There are various issues with this type of conversion. One problem is that the conventional conversion (1) is biased [5], [6]. A second is that the calculation of the converted measurement error covariance requires the true range and bearing, unavailable in practice. The practical resolution to this problem results in correlation between the covariance measurement error estimate (evaluated at the measurement) and the measurement error leading to an estimation bias when the converted measurement is used in tracking [8], [4], [2].

II. Evaluation of the Conversion Bias

Analysis of the expected value of the conventional conversion (1) shows that the conversion introduces a bias in the mean of the converted measurement [6]. The bias can be found by taking the expectation of the converted range and bearing measurements, r_m and α_m. If the range measurement noise, w_r, and bearing measurement noise, w_α, are uncorrelated, zero mean, and Gaussian with standard deviations of σ_r and σ_α respectively; the expected converted measurement is as follows:

$$E\left[(r+w_r)\cos(\alpha+w_\alpha)\right] = e^{-\sigma_\alpha^2/2} r\cos\alpha \tag{2}$$

$$E\left[(r+w_r)\sin(\alpha+w_\alpha)\right] = e^{-\sigma_\alpha^2/2} r\sin\alpha \tag{3}$$

Evident in (2) and (3) is that there is bias along the true bearing to the target with a magnitude of $r\left(e^{-\sigma_\alpha^2/2}-1\right)$. The absolute bias increases for long ranges and poor angle accuracies. The relative bias (i.e. the bias divided by the true range) is a function of angle accuracy. To compensate for the bias, previous authors proposed an Unbiased Converted Measurement (UCM) [6]:

$$x_m^{\mathrm{UCM}} = e^{\sigma_\alpha^2/2} r_m \cos\alpha_m \tag{4}$$

$$y_m^{\mathrm{UCM}} = e^{\sigma_\alpha^2/2} r_m \sin\alpha_m \tag{5}$$

III. Estimation of the Covariance

The true measurement covariance of the UCM is given by:

$$R_{true}^{11} = \frac{1}{2}\left(r^2+\sigma_r^2\right)\left[1+\cos(2\alpha)e^{-2\sigma_\alpha^2}\right]e^{\sigma_\alpha^2} - r^2\cos^2\alpha \tag{6}$$

$$R_{true}^{22} = \frac{1}{2}\left(r^2+\sigma_r^2\right)\left[1-\cos(2\alpha)e^{-2\sigma_\alpha^2}\right]e^{\sigma_\alpha^2} - r^2\sin^2\alpha \tag{7}$$

$$R_{true}^{12} = \frac{1}{2}\left(r^2+\sigma_r^2\right)\left[\sin(2\alpha)e^{-2\sigma_\alpha^2}\right]e^{\sigma_\alpha^2} - r^2\cos(\alpha)\sin(\alpha) \tag{8}$$

Since the true covariance requires the true range and bearing, it cannot be calculated in practice. Two approaches have been proposed to approximate the covariance using the

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measurements. The UCM approach evaluates the covariance at the measurements [6], namely,

$$R^{11}_{\text{UCM}} = \frac{1}{2}\left(r_m^2 + \sigma_r^2\right)\left[1 + \cos(2\alpha_m)e^{-2\sigma_\alpha^2}\right] + \left[e^{\sigma_\alpha^2} - 2\right] r_m^2 \cos^2 \alpha_m \tag{9}$$

$$R^{22}_{\text{UCM}} = \frac{1}{2}\left(r_m^2 + \sigma_r^2\right)\left[1 - \cos(2\alpha_m)e^{-2\sigma_\alpha^2}\right] + \left[e^{\sigma_\alpha^2} - 2\right] r_m^2 \sin^2 \alpha_m \tag{10}$$

$$R^{12}_{\text{UCM}} = \frac{1}{2}\left(r_m^2 + \sigma_r^2\right)\left[\sin(2\alpha_m)e^{-2\sigma_\alpha^2}\right] + \left[e^{\sigma_\alpha^2} - 2\right] r_m^2 \cos(\alpha_m)\sin(\alpha_m) \tag{11}$$

It can be seen that the measurement conversion (4) and (5) is derived by conditioning on the true range and bearing, while the error covariance (9) - (11) is derived by conditioning on the measurements. This incompatibility was pointed out by previous authors along with a modified unbiased conversion method [3]. The resulting Modified Unbiased Converted Measurement (MUCM), shown below, resolves the incompatibility, but results in a biased estimate.

$$x_m^{\text{MUCM}} = e^{-\sigma_\alpha^2/2} r_m \cos \alpha_m \tag{12}$$

$$y_m^{\text{MUCM}} = e^{-\sigma_\alpha^2/2} r_m \sin \alpha_m \tag{13}$$

$$R^{11}_{\text{MUCM}} = \frac{1}{2}\left(r_m^2 + \sigma_r^2\right)\left[1 + \cos(2\alpha_m)e^{-2\sigma_\alpha^2}\right] - e^{\sigma_\alpha^2} r_m^2 \cos^2 \alpha_m \tag{14}$$

$$R^{22}_{\text{MUCM}} = \frac{1}{2}\left(r_m^2 + \sigma_r^2\right)\left[1 - \cos(2\alpha_m)e^{-2\sigma_\alpha^2}\right] - e^{\sigma_\alpha^2} r_m^2 \sin^2 \alpha_m \tag{15}$$

$$R^{12}_{\text{MUCM}} = \frac{1}{2}\left(r_m^2 + \sigma_r^2\right)\left[\sin(2\alpha_m)e^{-2\sigma_\alpha^2}\right] - e^{\sigma_\alpha^2} r_m^2 \cos(\alpha_m)\sin(\alpha_m) \tag{16}$$

A comparison of the UCM and MUCM conversion is instructive. The advantage of the UCM conversion is that it is unbiased, an essential attribute in state estimation. The MUCM conversion, however, results in a lower mean squared error [2]. Both conversion techniques use a multiplicative term. The multiplicative term that results in the smallest expected square error can be derived using a factor η as follows:

$$x_m^{\text{MMSE}} = \eta r_m \cos(\alpha_m) \tag{17}$$

$$y_m^{\text{MMSE}} = \eta r_m \sin(\alpha_m) \tag{18}$$

The expected squared error is

$$\eta^2(r + \sigma_r^2) - 2\eta r^2 e^{-\sigma_\alpha^2/2} + r^2 \tag{19}$$

The minimizing η, shown below, requires knowledge of the true range, namely,

$$\eta = \frac{r^2}{r^2 + \sigma_r^2} e^{-\sigma_\alpha^2/2} \tag{20}$$

This term is bounded by the MUCM scaling term. Therefore, the mean square error of the MUCM conversion is always less than that of the UCM conversion [7]. In this light, the MUCM conversion can be viewed as a shrinkage technique that reduces the MSE at the expense of introducing a bias.

IV. Evaluation of the Estimation Bias

Both the UCM and MUCM conversions utilize the measurement to estimate the converted measurement error covariance. As a result, *the estimate of the covariance becomes correlated with the measurement noise*, leading to a *biased estimator* [8], [4], [2]. In order to analyze this phenomenon, a position estimator for a static target using converted measurements is considered. For convenience, a true bearing of 0° is used, resulting in a bias conveniently along the x-axis. The estimation bias, b_{est}, and overall bias, b_{tot} are defined, using the expectation operator E, as:

$$b_{est,\text{UCM}} = E\left[\hat{x}\right] - E\left[e^{\sigma_\alpha^2/2} r_m \cos \alpha_m\right] \tag{21}$$

$$b_{est,\text{MUCM}} = E\left[\hat{x}\right] - E\left[e^{-\sigma_\alpha^2/2} r_m \cos \alpha_m\right] \tag{22}$$

$$b_{tot} = E\left[\hat{x}\right] - x_{true} \tag{23}$$

While for the example geometry the bias is along the x-axis, in general it is along the true line of sight to the target. For this static estimation problem, consider the results of a linear least squares estimator (LLSE) using the UCM and MUCM conversion techniques. The LLSE is an average of the converted measurements, weighted by the inverse of the converted measurement noise covariance. Due to the dependence of the converted measurement noise covariance on the measurement noise, the estimator is biased.

As Fig. 1 shows, estimation bias due to the above correlation is a problem common to the UCM and MUCM conversion. It is interesting to note that while the estimation bias for UCM and MUCM are similar (Fig. 1), the overall bias (Fig. 2) is the smallest for the MUCM technique. This is due to the fact that the MUCM conversion bias and the estimation bias are in opposite directions. This explains why MUCM conversion has outperformed UCM conversion in tracking simulations. While the MUCM conversion bias and estimation bias seem to have a symbiotic relationship, using a biased conversion is not ideal for recursive estimation. For trackers using low process noise, the MUCM conversion will eventually converge to a solution with little bias. For higher process noise situations, where the tracker relies more on the converted measurement, the MUCM's biased measurement conversion will degrade performance.

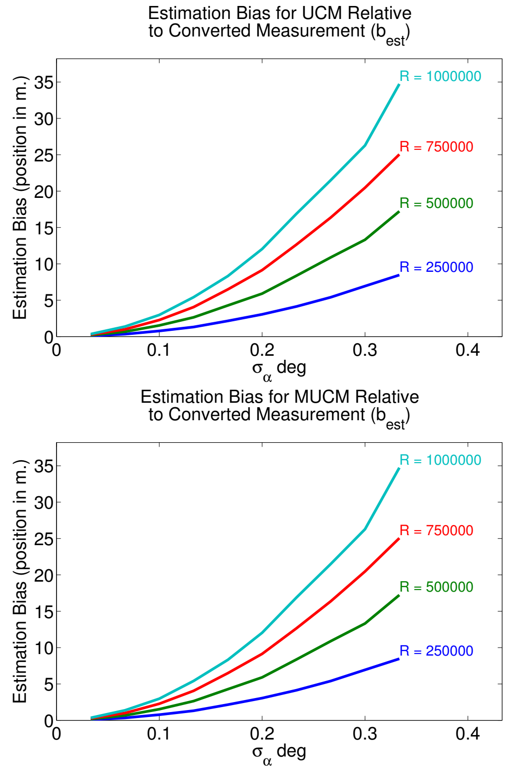

Fig. 1. Position estimation bias versus σ_α for a 10,000 sample LLSE using UCM and MUCM conversion methods with ranges of 250,000; 500,000; 750,000 and 1,000,000. For all cases σ_r = 0.5.

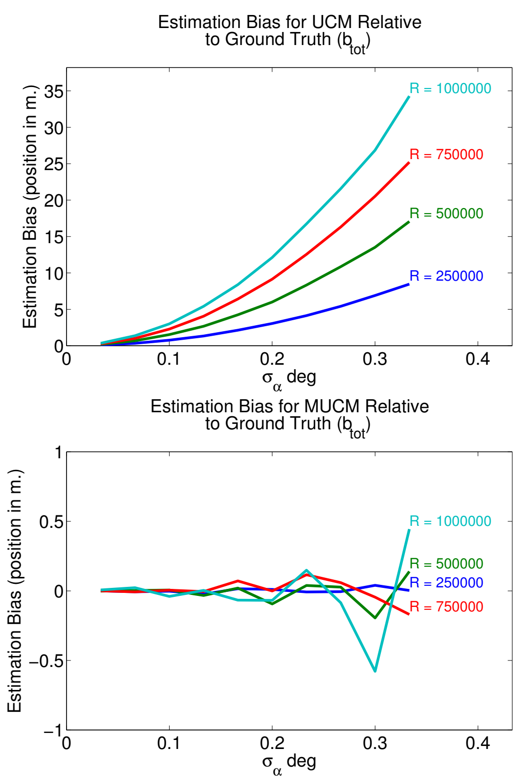

Fig. 2. Total position bias versus σ_α for a 10,000 sample LLSE using UCM and MUCM conversion methods with ranges of 250,000; 500,000; 750,000 and 1,000,000. For all cases σ_r = 0.5.

V. Decorrelated Unbiased Measurement Conversion

To overcome the disadvantages of the CMKF using the UCM and MUCM conversion techniques, the Decorrelated[1] Unbiased Converted Measurement (DUCM) has been proposed [2]. The design goals used in the development of this technique were:

1) Utilize an unbiased measurement conversion
2) Avoid correlation of the converted measurement covariance estimate and the measurement noise to preclude estimation bias
3) Provide minimum mean square error estimates.

To achieve the first goal, the UCM measurement conversion is used:

$$x_m^{\mathrm{DUCM}} = x_m^{\mathrm{UCM}} = e^{\sigma_\alpha^2/2} r_m \cos\alpha_m \tag{24}$$

$$y_m^{\mathrm{DUCM}} = y_m^{\mathrm{UCM}} = e^{\sigma_\alpha^2/2} r_m \sin\alpha_m \tag{25}$$

To decorrelate the estimation of the measurement covariance from the measurement noise, an approach similar to previous work is used [8]. While conditioning on the previous measurement has been proposed [8], the approach used here is to condition on the predicted estimate (i.e., use one-step predictions), namely,

$$\begin{aligned} R_{\mathrm{DUCM}}^{11} = {} & \frac{1}{2}\left(r_m^2 + \sigma_r^2 + \sigma_{r_t}^2\right) \\ & \cdot \left[1 + \cos(2\alpha_t) e^{-2\sigma_\alpha^2} e^{-2\sigma_{\alpha_t}^2}\right] e^{\sigma_\alpha^2} \\ & - \frac{1}{2}\left(r_m^2 + \sigma_{r_t}^2\right)\left[1 + \cos(2\alpha_t) e^{-2\sigma_{\alpha_t}^2}\right] \end{aligned} \tag{26}$$

$$\begin{aligned} R_{\mathrm{DUCM}}^{22} = {} & \frac{1}{2}\left(r_m^2 + \sigma_r^2 + \sigma_{r_t}^2\right) \\ & \cdot \left[1 - \cos(2\alpha_t) e^{-2\sigma_\alpha^2} e^{-2\sigma_{\alpha_t}^2}\right] e^{\sigma_\alpha^2} \\ & - \frac{1}{2}\left(r_m^2 + \sigma_{r_t}^2\right)\left[1 - \cos(2\alpha_t) e^{-2\sigma_{\alpha_t}^2}\right] \end{aligned} \tag{27}$$

$$\begin{aligned} R_{\mathrm{DUCM}}^{12} = {} & \frac{1}{2}\left(r_m^2 + \sigma_r^2 + \sigma_{r_t}^2\right) \\ & \cdot \left[\sin(2\alpha_t) e^{-2\sigma_\alpha^2} e^{-2\sigma_{\alpha_t}^2}\right] e^{\sigma_\alpha^2} \\ & - \frac{1}{2}\left(r_m^2 + \sigma_{r_t}^2\right)\left[\sin(2\alpha_t) e^{-2\sigma_{\alpha_t}^2}\right] \end{aligned} \tag{28}$$

where r_t and α_t are the predicted estimate's range and bearing and $\sigma_{\alpha_t}^2$ and $\sigma_{r_t}^2$ are their associated variances. Various techniques could be used for approximating these quantities; for example use of the Unscented Transform has been proposed [9]. The technique chosen here is a linearization of tracked covariance, ignoring correlation between range and bearing errors. Defining the predicted position as x_t and y_t and its associated covariance as:

[1] Some researchers prefer the name 'Federated', alluding to the union of decorrelating and debiasing.

$$\begin{bmatrix} P_{xx} P_{xy} \\ P_{yx} P_{yy} \end{bmatrix} \tag{29}$$

The predicted range and approximated range variance are as follows:

$$r_t = \sqrt{{x_t}^2 + {y_t}^2} \tag{30}$$

$$\sigma_{r_t}^2 = \begin{bmatrix} \frac{\partial r_t}{\partial x_t} \frac{\partial r_t}{\partial y_t} \end{bmatrix} \begin{bmatrix} P_{xx} P_{xy} \\ P_{yx} P_{yy} \end{bmatrix} \begin{bmatrix} \frac{\partial r_t}{\partial x_t} \\ \frac{\partial r_t}{\partial y_t} \end{bmatrix} \tag{31}$$

which simplifies to:

$$\sigma_{r_t}^2 = \frac{P_{xx} x_t^2 + 2P_{xy} x_t y_t + P_{yy} y_t^2}{x_t^2 + y_t^2} \tag{32}$$

Similarly, the predicted bearing and approximated bearing variance are:

$$\alpha_t = \tan^{-1}\left(\frac{y_t}{x_t}\right) \tag{33}$$

$$\sigma_{\alpha_t}^2 = \begin{bmatrix} \frac{\partial \alpha_t}{\partial x_t} \frac{\partial \alpha_t}{\partial y_t} \end{bmatrix} \begin{bmatrix} P_{xx} P_{xy} \\ P_{yx} P_{yy} \end{bmatrix} \begin{bmatrix} \frac{\partial \alpha_t}{\partial x_t} \\ \frac{\partial \alpha_t}{\partial y_t} \end{bmatrix} \tag{34}$$

which simplifies to:

$$\sigma_{\alpha_t}^2 = \frac{P_{xx} y_t^2 - 2P_{xy} x_t y_t + P_{yy} x_t^2}{(x_t^2 + y_t^2)^2} \tag{35}$$

Evaluation of the DUCM technique indicates that neither the estimation bias nor the overall bias are significant (Fig. 3).

While the DUCM technique has the advantage of unbiased conversion and negligible estimation bias, based on the arguments of (20), it will have a larger mean square error than the MUCM technique for the initial conversion and first few recursive estimates. To overcome this issue, a shrinkage technique is applied to the output of the filter. By applying the scaling factor only to the output of the filter, the Kalman Filter assumption of unbiased measurements is not violated. The scaling factor (36) converts the unbiased estimate into an approximate MMSE estimate.

$$\eta_{\mathrm{DUCM}} = e^{-\sigma_{\alpha_t}^2} \tag{36}$$

VI. Application to Converted Measurement Kalman Filter

The DUCM technique can be applied to the Converted Measurement Kalman Filter (CMKF) for improved state estimation. To evaluate realistic tracking performance, the general approach of previous works is adopted, with the appropriate values to be relevant to radar tracking. The target's initial x and y positions are taken from independent draws from a Gaussian distribution with mean 500,000 m and standard deviation of 10,000 m. Target speed is taken from a Gaussian distribution with a mean of 75 m/s and a standard deviation of 10 m/s. Target heading is taken from a uniform distribution. The target follows a constant velocity track and is estimated using a nearly constant velocity tracker with a discrete white noise acceleration model [1]. One point initialization of the tracker is used with an initial velocity estimate of 0 m/sec and standard deviation of 47.5 m/s in each component.

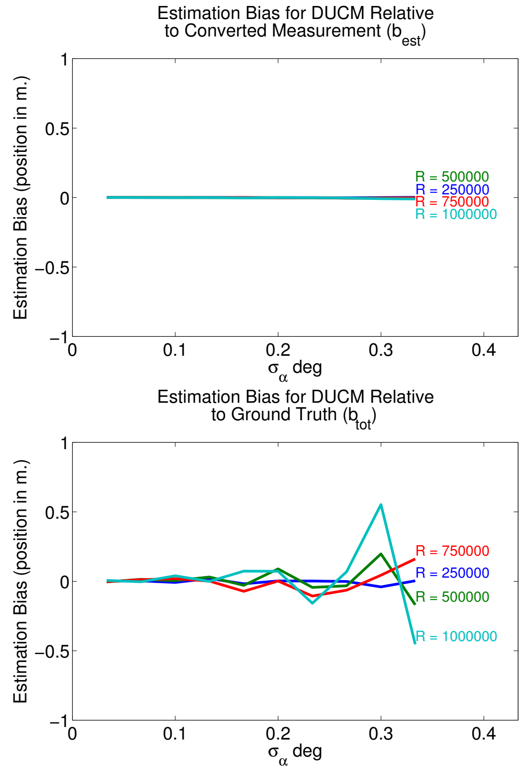

Fig. 3. Estimation and total position bias versus σ_α for a 10,000 sample LLSE using DUCM conversion method with ranges of 250,000; 500,000; 750,000 and 1,000,000. For all cases σ_r = 0.5.

A. Mean Square Error performance

Evaluation of tracking performance indicates that the CMKF using the DUCM technique outperforms the CMKF using UCM or MUCM in position MSE. CMKF-DUCM velocity MSE slightly underperforms during the initial scans, but has the best performance for later scans. Fig. 4 and 5 show the MSE comparison of the three techniques.

B. Average Normalized Estimation Error Squared (ANEES) performance

To ensure credibility of the DUCM method, the ANEES performance is examined. The ANEES scaled to the state dimension, *n*, is [1]

$$\mathrm{ANEES} = \frac{1}{Nn} \sum_{i=1}^{N} \tilde{X}_i^T P_i^{-1} \tilde{X}_i \tag{37}$$

where $\tilde{X}_i$ is the estimation error and P_i is the error covariance for trial i. The ANEES of a consistent estimator should be close to 1. Fig. 6 shows that the DUCM approach is the most consistent based on the ANEES.

VII. Conclusion

When using converted measurements in tracking, two sources of bias need to be evaluated and eliminated. The first is measurement conversion bias that occurs when the conversion process introduces a bias in the mean of the

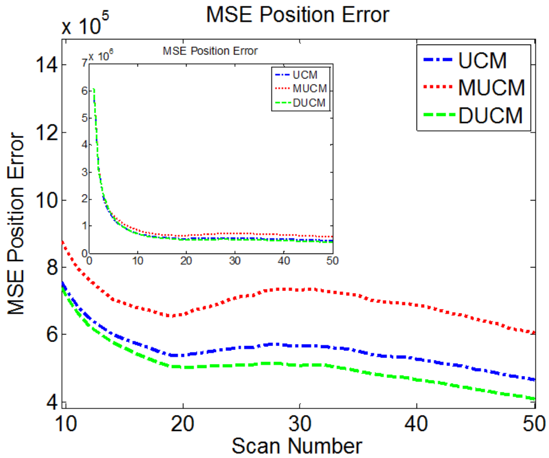

Fig. 4. CMKF position MSE comparison for the UCM, MUCM and DUCM conversion methods from 10,000 Monte-Carlo runs. The range and bearing measurement noises used were uncorrelated Gaussian with σ_r = 0.5m and $\sigma_\alpha = 3/15°$. The inset figure shows the results of 50 scans, while the main figure is zoomed into scans 10 to 50.

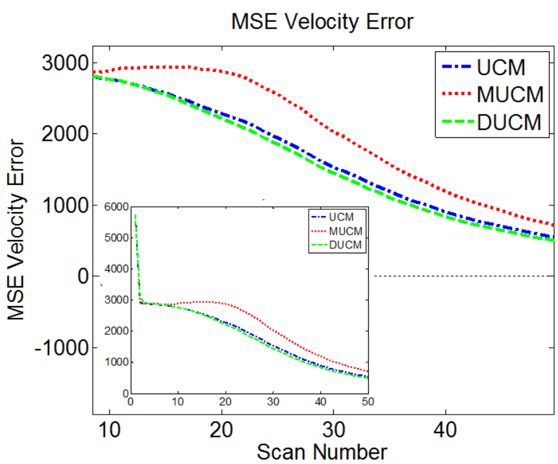

Fig. 5. CMKF velocity MSE for the UCM, MUCM and DUCM conversion methods from 10,000 Monte-Carlo runs. The range and bearing measurement noises used were uncorrelated Gaussian with σ_r = 0.5m and $\sigma_\alpha = 3/15°$. The inset figure shows the results of 50 scans, while the main figure is zoomed into scans 10 to 50.

converted measurement. The second source of bias is estimation bias that occurs when the estimate of the measurement covariance is correlated with the converted measurement noise, leading to a biased Kalman gain. It has been shown for converted measurement tracking problems that a decorrelated version of the Unbiased Measurement Conversion (DUCM) exhibits improved performance over the previously proposed techniques. Future work may include evaluation of the DUCM

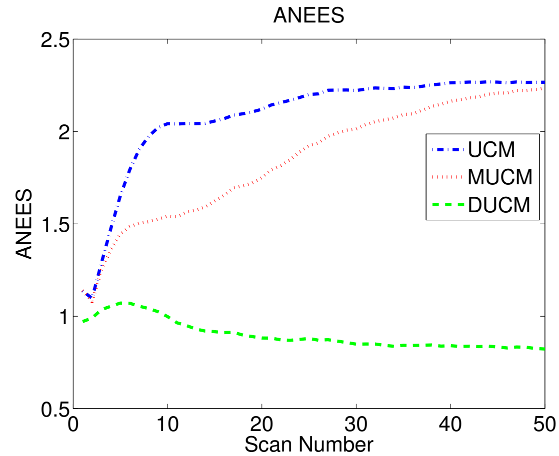

Fig. 6. ANEES comparison for the UCM, MUCM and DUCM conversion methods from 10,000 Monte-Carlo runs. The range and bearing measurement noises used were uncorrelated Gaussian with σ_r = 0.5m and $\sigma_\alpha = 3/15°$.

technique in additional scenarios and extension from polar to spherical coordinate conversion.

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